Chapter 16

Simulation Modeling

Set 16.1a

1

R1	R2	X	Y	(X-1)^2+(Y-2)^2	1=in, 0=out
0.0589	0.6733	-3.411	3.733	22.46021	1
0.4799	0.9486	0.799	6.486	20.164597	1
0.6139	0.5933	2.139	2.933	2.16781	1
0.9341	0.1782	5.341	-1.218	29.199805	0
0.3473	0.5644	-0.527	2.644	2.746465	1
0.3529	0.3646	-0.471	0.646	3.997157	1
0.7676	0.8931	3.676	5.931	22.613737	1
0.3919	0.7876	-0.081	4.876	9.439937	1
0.5199	0.6358	1.199	3.358	1.883765	1
0.7472	0.8954	3.472	5.954	21.7449	1
				Total=	9
				Area estimate=	90

Exact area = 78.54 cm2. Estimate from Figure 18-2 = 78.56 cm2 for a sample size of n=30,000. Current estimate = 90 cm2, which is unreliable because the sample size is too small.

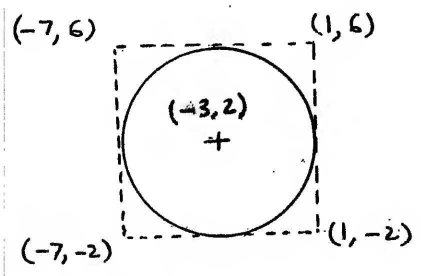
2

(a) $x = -7 + 8R_1$

$y = -2 + 8R_2$

$f(x) = 1/8, \quad -7 \le x \le 1$

$f(y) = 1/8, \quad -2 \le x \le 6$



(b)

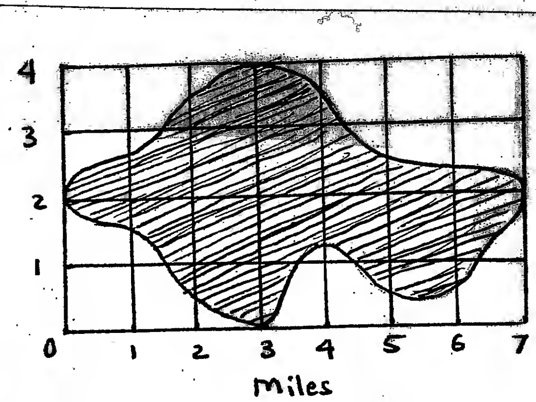
Monte Carlo Estimation of the Area of a Circle

Input data			
Nbr. Replications, N =	10		
Sample size, n =	100,000	Steps =	1
Radius, r =	4		
Center, cx =	-3		
Center, cy =	2		
Output results			
Exact area =	50.265		
Press to Execute Monte Carlo			

Monte Carlo Calculations:

	n=100000
Replication 1	50.223
Replication 2	50.378
Replication 3	50.113
Replication 4	50.260
Replication 5	50.244
Replication 6	50.330
Replication 7	50.327
Replication 8	50.252
Replication 9	50.236
Replication 10	50.467
Mean =	50.283
Std. Deviation =	0.099
95% lower conf. limit =	50.212
95% upper conf. limit =	50.354

3



R_1	R_2	x	y	in?
.0589	.6733	.4123	2.6932	No
.4799	.9486	3.3593	3.7944	Yes
.6139	.5933	4.2973	2.3732	Yes
.9341	.1782	6.5387	.7128	NO
.3473	.5644	2.4311	2.2576	Yes
.3529	.3646	2.4703	1.4584	Yes
.7676	.8931	5.3732	3.5724	No
.3919	.7876	2.7433	3.1504	Yes
.5199	.6358	3.6393	2.5432	No
.7472	.8954	5.2304	3.5816	No

points in = 5

Area estimate $= \frac{5}{10} \times (4 \times 7) = 14$ miles^2

4

(a) $P\{H\} = .5 \qquad P\{T\} = .5$

If $0 \le R \le .5$, Jim gets \$10

$.5 < R \le 1$, Jan gets \$10

(b)

R	Jan's pay	R	Jan's pay
.0589	-10	.3529	-10
.6733	10	.3646	-10
.4799	-10	.7676	10
.9486	10	.8931	10
.6139	10	.3919	-10
.5933	10	.7876	10
.9341	10	.5199	10
.1782	-10	.6358	10
.3473	-10	.7472	10
.5644	10	.8954	10
$\bar{X}_1 = \$2$		$\bar{X}_2 = \$4$	

continued...

R	Jan's pay	R	Jan's pay	R	Jan's pay
.5861	10	.3455	-10	.7900	10
.1281	-10	.4871	-10	.7698	10
.2867	-10	.8111	10	.2871	-10
.8216	10	.8912	10	.9534	10
.3866	-10	.4291	-10	.1394	-10
.7125	10	.2302	-10	.9025	10
.2108	-10	.5423	10	.1605	-10
.3575	-10	.4208	-10	.3567	-10
.2926	-10	.6975	10	.3070	-10
.8261	10	.5954	10	.5513	10
$\bar{X}_3 = -\$2$		$\bar{X}_4 = \$0$		$\bar{X}_5 = \$0$	

(b) Av. Jan's pay based on 5 repls.

$$= \frac{2+4-2+0+0}{5}$$

$$= \$.8$$

$$S = \sqrt{\frac{(2-.8)^2+(4-.8)^2+(-2-.8)^2+2(0-.8)^2}{5-1}}$$

$$= \sqrt{\frac{80.8}{4}} = 2.28$$

Confidence interval:

$$.8 - \frac{2.28}{\sqrt{5}}\, t_{.025,4} \le \mu \le .8 + \frac{2.28}{\sqrt{5}}\, t_{.025,4}$$

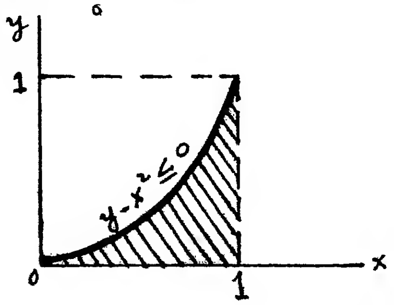
Given $t_{.025,4} = 2.776$, the 95% confidence interval is

$$-2.03 \le \mu \le 3.63$$

(c) Theoretical Jan's payoff = \$0.

5

Estimate $\int_0^1 x^2\,dx$



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(a) Let x=R1 and y=R2.
Experiment: If R2<R1^2, count point "in".
Estimate of integral = (1x1)(Points "in")/5

(b)

	R1	R2	1=in, 0=out
Rep 1	0.0589	0.6733	0
	0.4799	0.9486	0
	0.6139	0.5933	0
	0.9341	0.1782	1
	0.3473	0.5644	0
	Integral estimate =		0.2
Rep 2	0.3529	0.3646	0
	0.7676	0.8931	0
	0.3919	0.7876	0
	0.5199	0.6358	0
	0.7472	0.8954	0
	Integral estimate =		0
Rep 3	0.5869	0.1281	1
	0.2867	0.8216	0
	0.8261	0.3866	1
	0.7125	0.2108	1
	0.3575	0.2926	0
	Integral estimate =		0.6
Rep 4	0.3455	0.4871	0
	0.8111	0.8912	0
	0.4291	0.2302	0
	0.5954	0.5423	0
	0.4208	0.6975	0
	Integral estimate =		0
overall integral estimate =			0.2
Std. Deviation =			0.244949
95% lower confidence limit =			-0.189714
95% upper confidence limit =			0.5485706
Exact integral value =			0.3333

The given estimate is not "good" when compared with the exact value because sample size (n = 5) is too small.

6

7 ≡ (6,1), (5,2), (4,3), (3,4), (2,5), (1,6)
11 ≡ (6,5), (5,6)

Monte Carlo experiment:

R	outcome
$0 \le R \le 1/6$	1
$1/6 < R \le 1/3$	2
$1/3 < R \le 1/2$	3
$1/2 < R \le 2/3$	4
$2/3 < R \le 5/6$	5
$5/6 < R \le 1$	6
$0 \le R \le .167$	1
$.167 < R \le .333$	2
$.333 < R \le .5$	3
$.5 < R \le .667$	4
$.667 < R \le .833$	5
$.833 < R \le 1$	6

Continued...

Set 16.1a

R_1	R_2	Sum	Payoff
.0589	.6733	$1+5=\boxed{6}$ point	
.4799	.9486	$3+6=9$	
.6139	.5933	$4+4=8$	
.9341	.1782	$6+2=8$	
.3473	.5644	$3+4=7 \rightarrow$	$-\$10$
.3529	.3646	$3+3=\boxed{6}$ point	
.7676	.8931	$5+6=11$	
.3919	.7876	$3+5=8$	
.5199	.6358	$4+4=8$	
.7472	.8954	$5+6=11$	
.5869	.1281	$4+1=5$	
.2867	.8216	$2+5=7 \rightarrow$	$-\$10$
.8261	.3866	$5+3=\boxed{8}$ point	
.7125	.2108	$5+2=7 \rightarrow$	$-\$10$
.3575	.2926	$3+2=\boxed{5}$ point	
.3455	.4871	$3+3=6$	
.8111	.8912	$5+6=11$	
.4291	.2302	$3+2=5 \rightarrow$	$\$10$
.5954	.5423	$4+4=\boxed{8}$ point	
.4208	.6975	$3+5=8 \rightarrow$	$\$10$

7

Lead time:

$0 \le R \le .5, \quad L = 1$ day

$.5 < R \le 1, \quad L = 2$ days

Demand/day:

$0 \le R \le .2, \quad d = 0$ unit

$.2 < R \le .9, \quad d = 1$ unit

$.9 < R \le 1. \quad d = 2$ units

Let $p(d, L)$ be the joint pdf of demand and lead time. The procedure calls for constructing a frequency table of demand and lead time. The maximum demand during lead time is $2 \times 2 = 4$ units, so that the demand $d = 0, 1, 2, 3, 4$. We will use the random numbers in Table 16-1 in the following manner: First use a random number to generate a lead time. If $L=1$ day, use one

continued...

7 continued

random number to generate the demand in that day. If $L=2$ days, use two random numbers to generate the demands for the two days. For example, $R=.058962$ yields $L=1$. Next, $R=.6733$ gives $d=1$. Thus, we update the frequency table by increasing the frequency of the entry ($d=1$, $L=1$) by one. The frequency table using the first two columns of R in Table 16-1 is

L \ d	0	1	2	3	4
1	I	~~IIII~~ II	II	0	0
2	II	0	~~IIII~~ II	IIII	0

L \ d	0	1	2	3	4
1	1	7	2	0	0
2	2	0	7	4	0

Total $n = 23$

Relative frequency table:

L \ d	0	1	2	3	4	P(L)
1	1/23	7/23	2/23	0	0	10/23
2	2/23	0	7/23	4/23	0	13/23
p(d)	3/23	7/23	9/23	4/23	0	

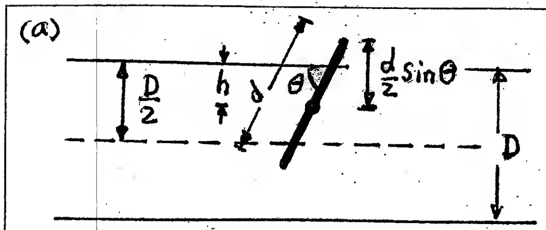
Notice that

$$p(d) = \sum_L p(d, L)$$

$$p(L) = \sum_d p(d, L)$$

8

(a)



From graph, needle will touch line or cross it if

$$h \le \frac{d}{2}\sin\theta$$

(b) Generate $h = R_1 \times D/2$

$$\theta = \pi \times R_2$$

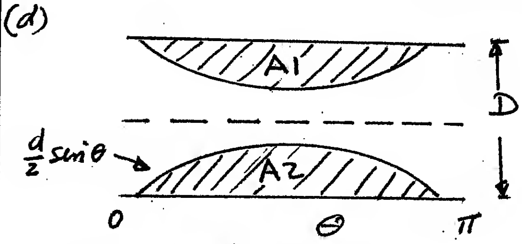
If $h \le \frac{d}{2}\sin\theta$, needle touches. Else it doesn't.

Probability estimate $= \dfrac{\#\text{ touches}}{\text{sample size}}$

(c)

A	B	C	D	E
	D=	20	d=	10
	(RAND()*C1)*0.5	RAND()*PI()	E1*0.5*SIN(C4)	IF(B4<=D4,1,0)
	h	theta	d*sin(theta)/2	1=touch, 0=else
Rep 1	8.396953573	1.3165558	4.839272983	0
	7.107859045	2.9048959	1.172463622	0
	0.27542965	0.8440783	3.736795168	1
	1.267504547	2.8354706	1.506816139	1
	9.237262421	0.7436482	3.38488765	0
	2.495379696	2.9719552	0.844125326	0
	4.253169953	2.8396976	1.486650397	0
	8.516662244	1.4161445	4.940326141	0
	4.224254495	0.7887632	3.547410981	0
	3.690266876	3.0811599	0.301979787	0
		Estimate of probability=		0.2
Rep 2	0.712918949	1.5238102	4.994481772	1
	9.381794079	2.5979258	2.586388239	0
	1.360072144	2.0189288	4.506289193	1
	8.477675064	1.9724771	4.60202594	0
	0.99443686	1.300734	4.81877136	1
	5.170438974	1.4568612	4.967582038	0
	5.056822846	1.6844549	4.967739087	0
	5.864264693	0.0683356	0.341412027	0
	6.87137267	2.6283793	2.454895584	0
	1.092023022	2.6522347	2.350296303	1
		Estimate of probability=		0.4
Rep3	9.712756211	1.694489	4.961799031	0
	6.686447356	1.2243834	4.702983326	0
	6.436673778	2.4581589	3.157296664	0
	1.324134345	2.2441568	3.908652279	1
	1.775706228	2.255079	3.874363448	1
	0.090587765	2.7080167	2.100592855	1
	4.979938633	2.5138689	2.936520016	0
	8.678634219	2.7348178	1.978247037	0
	2.179672677	1.8339609	4.827857959	1
	9.640572895	1.2431615	4.734030551	0
		Estimate of probability=		0.4
Rep 4	8.227016322	2.6999829	2.136976805	0
	8.757368267	2.1537385	4.174233356	0
	4.203914479	0.1860064	0.92467824	0
	6.098369885	2.1672345	4.13670754	0
	4.960185836	0.7841548	3.531135292	0
	3.899078191	1.8047989	4.863730557	1
	5.840727605	0.727722	3.325852126	0
	6.645324046	0.498725	2.391531067	0
	5.361422671	0.89898	3.91346242	0
	3.223016816	1.6715052	4.974665749	1
		Estimate of probability=		0.2
			Mean value =	0.3
			Std. Deviation =	0.1155
			95% LCL =	0.1163
			95%UCL =	0.4837

(d)



$$\text{Exact probability} = \frac{A_1 + A_2}{\pi D}$$

$$= \frac{2\int_0^{\pi} \frac{d}{2}\sin\theta\, d\theta}{\pi}$$

$$= \frac{2d}{\pi D}$$

(e) From (c),

$$\tilde{p} = .3$$

Thus,

$$\frac{2d}{\pi D} = .3$$

or

$$\pi \simeq \frac{2d}{.3D}$$

$$\simeq \frac{2 \times 10}{.3 \times 20}$$

$$\simeq 3.33$$

Set 16.2a

1

(a) Discrete

(b) Continuous

(c) Discrete

2

In discrete simulation, there are two main events: arrivals and departures. An arrival event may experience delay before starting service. When service has been completed, customer leaves the facility.

The description of the discrete simulation situation by arrival and departure events is the reason discrete simulation is associated with queues.

1

Events:

A_1 = rush job arrives

A_2 = regular job arrives

D_1 = rush job departs

D_2 = regular job departs

2

A_0 = job arrives at carousel

A_1 = job arrives at station 1

A_2 = job arrives at station 2

A_3 = job arrives at station 3

D_1 = job departs station 1

D_2 = job departs station 2

D_3 = job departs station 3

3

A_1 = car enters lane 1

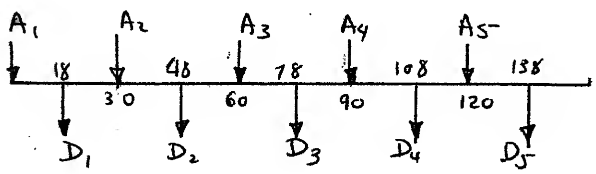
A_2 = car enters lane 2

A_3 = car goes elsewhere

D_1 = car departs lane 1

D_2 = car departs lane 2.

4

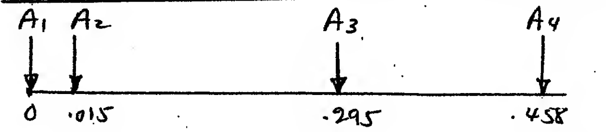


1

$$t = -\frac{1}{\lambda}\ln(1-R)$$

$\lambda = 4$ customers/hr

Customer	R	t(hrs)	Arrival time
1	—	—	0
2	.0589	.015	0+.015 = .015
3	.6733	.280	.015+.28 = .295
4	.4799	.163	.295+.163 = .458



2

$$f(t) = \frac{1}{b-a}, \quad a \le t \le b$$

$$F(t) = \int_0^t \frac{1}{b-a}\,dx = \frac{t-a}{b-a}, \quad a \le t \le b$$

$$R = \frac{t-a}{b-a}$$

$$t = a + (b-a)R$$

3

$f_1(t_1) = .5e^{-.5t}$, $\lambda = 1/2$ arrival/hr

$f_2(t) = \frac{1}{.9}$, $1.1 < t < 2$

$R = .0589$, $a_1 = -2\ln(1-.0589) = .12$ hr

$R = .6733$, $d_1 = 1.1 + .9 \times .6733 = 1.71$ hrs

$R = .4799$, $a_2 = -2\ln(1-.4799) = 1.31$ hrs

$R = .9486$, $a_3 = -2\ln(1-.9486) = 5.94$ hrs

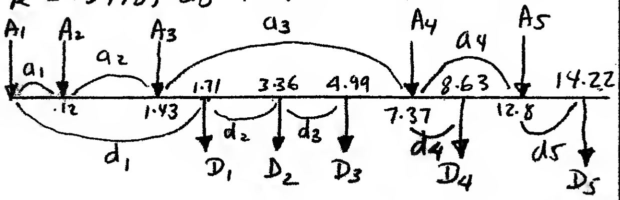
$R = .6139$, $d_2 = 1.1 + .9 \times .6139 = 1.65$ hrs

$R = .5933$, $d_3 = 1.1 + .9 \times .5933 = 1.63$ hrs

$R = .9341$, $a_4 = -2\ln(1-.9341) = 5.44$ hrs

$R = .1782$, $d_4 = 1.1 + .9 \times .1782 = 1.26$ hrs

$R = .3473$, $d_5 = 1.1 + .9 \times .3473 = 1.41$ hrs



4

(a)
$0 \le R < .2$, $d = 0$
$.2 \le R < .5$, $d = 1$
$.5 \le R < .9$, $d = 2$
$.9 \le R \le 1.$, $d = 3$

(b)

Day	R	Demand d	Stock level
0	—	—	5
1	.0589	0	5
2	.6733	2	3
3	.4799	1	2

Replenish stock on day 3

5

Repair/.2, Package/.8:

$0 \le R < .2$, go to Repair
$.2 \le R \le 1.$, go to Package

Package/.8, Repair/.2:

$0 \le R < .8$, go to Package
$.8 \le R \le 1$, go to Repair

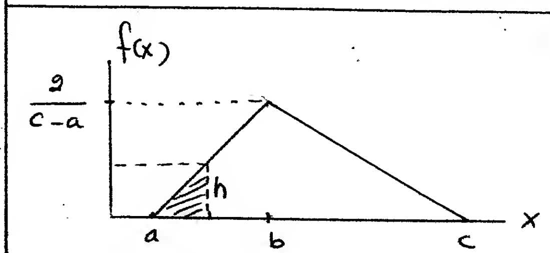
Example: $R = .1$ leads to Repair in the first case and to Package in the second case

6

$0 \le R < .5$: H
$.5 \le R \le 1.$: T

n	R	outcome	Payoff
1	.0589	H	\$2
1	.6733	T	0
2	.4799	H	$2^2 = \$4$

7



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7 continued

(a)
$$F(x)=\begin{cases}\dfrac{(x-a)^2}{(b-a)(c-a)}, & a\le x\le b\\[2ex] 1-\dfrac{(c-x)^2}{(c-b)(c-a)}, & b\le x\le c\end{cases}$$

For $R=\dfrac{(x-a)^2}{(b-a)(c-a)}$,

$x=a+\sqrt{R(b-a)(c-a)}$, $0\le R\le \dfrac{b-a}{c-a}$

For $R=1-\dfrac{(c-x)^2}{(c-b)(c-a)}$,

$x=c-\sqrt{(c-b)(c-a)(1-R)}$, $\dfrac{b-a}{c-a}\le R\le 1$

(b) $a=1$, $b=3$, $c=7$

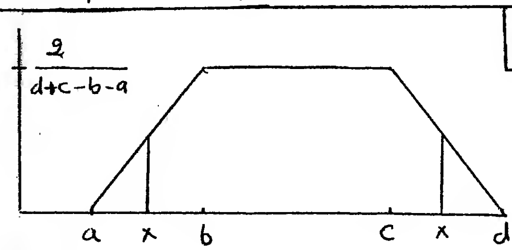
$$\frac{b-a}{c-a}=\frac{3-1}{7-1}=.333$$

Thus,

$$x=\begin{cases}1+\sqrt{(3-1)(7-1)R}=1+\sqrt{12R}, & 0\le R\le .333\\[2ex] 7-\sqrt{(7-3)(7-1)(1-R)}=7-\sqrt{24(1-R)}, & .333\le R\le 1\end{cases}$$

R	x
.0589	1.84
.6733	4.20
.4799	3.47
.9486	5.89
.6139	3.96

8



(a)
$$F(x)=\begin{cases}\dfrac{(x-a)^2}{(b-a)(d+c-b-a)} & a\le x\le b\\[2ex] \dfrac{1}{(b-a)(d+c-b-a)}+\dfrac{2(x-a)}{(d+c-b-a)} & b\le x\le c\\[2ex] 1-\dfrac{(d-x)^2}{(d-c)(d+c-b-a)} & c\le x\le d\end{cases}$$

Continued...

$R=\dfrac{(x-a)^2}{(b-a)(d+c-b-a)}$ gives

$x=a+\sqrt{(b-a)(d+c-b-a)R}$, $0\le R\le\dfrac{b-a}{(d+c-b-a)}$

$R=\dfrac{1}{(b-a)(d+c-b-a)}+\dfrac{2(x-b)}{(d+c-b-a)}$ gives

$x=\frac{1}{2}\left(R-\dfrac{1}{(b-a)(d+c-b-a)}\right)(d+c-b-a)$,

$\dfrac{b-a}{d+c-b-a}\le R\le 1-\dfrac{d-c}{(d+c-b-a)}$

$R=1-\dfrac{(d-x)^2}{(d-c)(d+c-b-a)}$

$x=d-\sqrt{(d-c)(d+c-b-a)(1-R)}$,

$1-\dfrac{d-c}{(d+c-b-a)}\le R\le 1$

(b) $a=1$, $b=2$, $c=4$, $d=6$

$1+\sqrt{(2-1)(6+4-2-1)R}=1+\sqrt{7R}$, $0\le R\le .143$

$2+\dfrac{6+4-2-1}{2}\left(R-\dfrac{1}{(2-1)(6+4-2-1)}\right)$
$=2+3.5(R-.143)$,
$.143\le R\le .714$

$6-\sqrt{(6-4)(6+4-2-1)(1-R)}$
$=6-\sqrt{14(1-R)}$
$.714\le R\le 1$

R	x
.0589	1.64
.6733	3.86
.4799	3.18
.9486	5.15
.6139	3.65

9

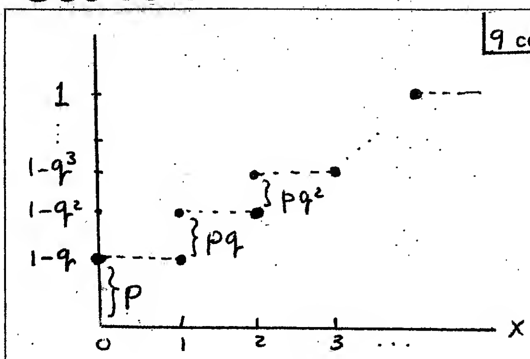
$f(x)=pq^x$, $x=0,1,2,\ldots$
$(p+q)=1$

$F(x)=p\sum_{t=0}^{x} q^t$

$=1-q^{x+1}$, $x=0,1,2,\ldots$

Continued...

9 continued



Sampling procedure:

if $0 \le R \le p$, then $x = 0$.

For $p < R \le 1$, we have

$$1-q^n \le R \le 1-q^{n+1}$$

or

$$n \le \frac{\ln(1-R)}{\ln q} \le n+1$$

Thus, for $p \le R \le 1$, compute

$$x = \left[\frac{\ln(1-R)}{\ln q}\right]$$

where $[a]$ is the largest integer less than or equal to a.

For $p = .6$, $q = .4$, we have

R	$\frac{\ln(1-R)}{\ln q}$	x
.0589	—	0
.6733	1.22	1
.4799	—	0
.9486	3.24	3
.6139	1.03	1

10

$$f(x) = \alpha\beta^{-\alpha} x^{\alpha-1} e^{-(x/\beta)^\alpha}, \quad x > 0$$

$$= \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1} e^{-\left(\frac{x}{\beta}\right)^\alpha}, \quad x > 0$$

$$F(x) = 1 - e^{-\left(\frac{x}{\beta}\right)^\alpha}, \quad x > 0$$

Thus,

$$R = 1 - e^{-\left(\frac{x}{\beta}\right)^\alpha}$$

or

$$x = \beta\left[-\ln(1-R)\right]^{1/\alpha}$$

1

$$y = -\frac{1}{5}\ln\{(.0589 \times .6733 \times .4799 \times .9486\}$$
$$= .803 \text{ hour}$$

2

$\lambda = 5$ events/hr, $t = 1$

$e^{-5\times 1} = e^{-5} = .00673$

i	$R_1R_2\cdots R_i$		
1	.0589		
2	$.0589 \times .6733$	=	.0397
3	$.0397 \times .4799$	=	.0190
4	$.0190 \times .9486$	=	.0181
5	$.0181 \times .6139$	=	.0111
6	$.0111 \times .5933$	=	.00656
7	$.00656 \times .9341$	=	.00614

Hence $n = 6$

3

$\mu = 8$, $\sigma = 1$, $N(8,1)$

Convolution method:

$$x = R_1 + R_2 + \cdots + R_{12} = 6.1094$$
$$y = 8 + 1(6.1094 - 6) = 8.1094$$

Box-Miller method:

$$x = \sqrt{-2\ln R_1}\cos(2\pi R_2)$$
$$= \sqrt{-2\ln .0589}\cos(2\pi \times .6733)$$
$$\cong -1.103$$

$$y = 8 + 1(-1.103) = 6.897$$

4

$\lambda = 6$/day $\quad n = 5$

$$y = -\frac{1}{6}\ln(.0589 \times .6733 \times .4799 \times .9486 \times .6139) = .751 \text{ hour}$$

5

$N(27,3)$: $\mu = 27$, $\sigma = 3$

Given R_1 and R_2, we have

$x_1 = \sqrt{-2\ln R_1}\cos(2\pi R_2)$

$x_2 = \sqrt{-2\ln R_1}\sin(2\pi R_2)$

$y_1 = \mu + \sigma x_1$

$y_2 = \mu + \sigma x_2$

Continued...

	J	K	L	M	N	O
	Mean = 27		Std. Dev.= 3			
	R1	R2	x1	x2	y1	y2
5	0.0589	0.6733	-1.1030306	-2.108827	23.69091	20.67352
6	0.4799	0.9486	1.149111	-0.384576	30.44733	25.84627
7	0.6139	0.5933	-0.8229152	-0.546495	24.53125	25.36051
				mean y=	25.09163	
				Sy	3.197533	

Formulas:

```
L5=    SQRT(-2*LN(J5))*COS(2*PI()*K5)
M4=    SQRT(-2*LN(J5))*SIN(2*PI()*K5)
N4=    $K$1+L4*$M$1
O4=    $K$1+M4*$M$1
```

6

$$x_i = 10 + (20-10)R_i$$
$$= 10 + 10R_i, \quad i = 1,2,3,4$$
$$t = x_1 + x_2 + x_3 + x_4$$
$$= 40 + 10(R_1 + R_2 + R_3 + R_4)$$

7

	R_1	R_2	R_3	R_4	t (sec)	Σt
1	.0589	.6733	.4799	.9486	61.61	61.60
2	.6139	.5933	.9341	.1782	63.20	124.81
3	.3473	.7676	.8931	.3919	64.00	188.81
4	.7876	.5199	.6358	.7472	66.91	255.72
5	.8954	.5869	.1281	.2867	58.94	314.69

The number of mice that exit the maze in 300 seconds is 4

8

Let $x_1, x_2, \ldots, x_r$ be r successive random deviates obtained from the geometric distribution as given in Problem 9, Set 18.3b. Then

$$x_i = \left[\frac{\ln R_i}{\ln(1-p)}\right], \quad i = 1,2,\ldots,r$$

Because the negative binomial is the convolution of r independent geometric random variables, it follows that a random negative binomial sample can be determined as

$$x = \sum_{i=1}^{r}\left[\frac{\ln R_i}{\ln(1-p)}\right]$$

Note that $[a]$ represents the largest integer $\le a$

1

Step 1: $R = .6139$

$X = .6139$

Step 2: $R = .5933$

Step 3: $\dfrac{f(.6139)}{g(.6139)} = .948 > .5933$ Reject X

Step 1: $R = .9341$, $X = .9341$

Step 2: $R = .1782$

Step 3: $\dfrac{f(.9341)}{g(.9341)} = \dfrac{.3693}{1.5} = .246 > .1782$ Reject X

Step 1: $R = .3473$, $X = .3473$

Step 2: $R = .5644$

Step 3: $\dfrac{f(.3473)}{g(.3473)} = .9067 > .5644$ Reject X

Step 1: $R = .3529$, $X = .3529$

Step 2: $R = .3646$

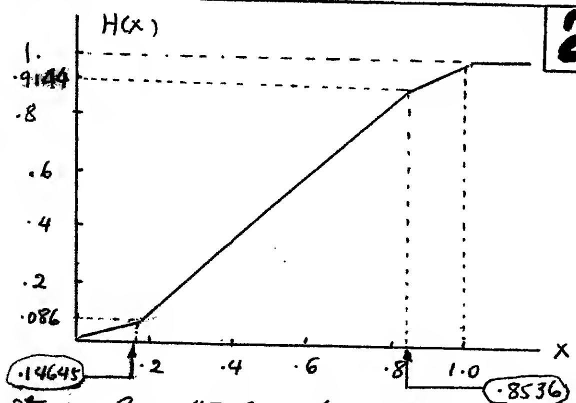
Step 3: $\dfrac{f(.3529)}{g(.3529)} = .913 > .3646$ Reject X

Step 1: $R = .7676$, $X = .7676$

Step 2: $R = .8931$

Step 3: $\dfrac{f(.7676)}{g(.7676)} = .7135 < .8931$ accept $X = .7676$

2



Step 1: $R = .4799$, $X = .4831$

Step 2: $R = .9486$

Step 3: $\dfrac{f(.4831)}{g(.4831)} = .9988 > .9486$ Reject X

Step 1: $R = .6139$, $X = .5974$

Step 2: $R = .5933$

continued...

2 continued

Step 3: $\dfrac{f(.5974)}{g(.5974)} = .962 > .5933$ reject X

Step 1: $R = .9341$ $X = .8804$

Step 2: $R = .1782$

Step 3: $\dfrac{f(.8804)}{g(.8804)} = .842 > .1782$ Reject X

Step 1: $R = .3529$, $X = .375$

Step 2: $R = .3646$

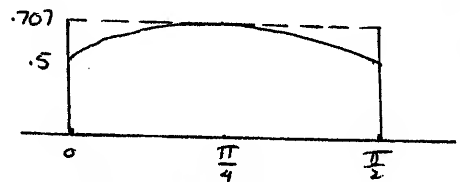
Step 3: $\dfrac{f(.375)}{g(.375)} = .937 > .3646$ Reject X

Step 1: $R = .7676$, $X = .7286$

Step 2: $R = .8931$

Step 3: $\dfrac{f(.7286)}{g(.7286)} = \dfrac{1.186}{1.5} = .791 < .8931$ accept X

3



$f(x) = \dfrac{\sin(x) + \cos(x)}{2}$ $\quad 0 \le x \le \frac{\pi}{2}$

$\max f(x) = .707$ at $x = \frac{\pi}{4}$

$g(x) = .707$ $\quad 0 \le x \le \pi/2$

$h(x) = \dfrac{g(x)}{\text{area under } g(x)}$

$= \dfrac{.707}{.707 \times \frac{\pi}{2}} = .637$ $\quad 0 \le x \le \frac{\pi}{2}$

4

$\int_{12}^{20} \dfrac{K_1}{t}\,dt = K_1 \ln \dfrac{20}{12} = 1$

Thus, $K_1 = 1.96$

$\int_{18}^{22} \dfrac{K_2}{t^2}\,dt = K_2\left(\frac{1}{18} - \frac{1}{22}\right) = 1$

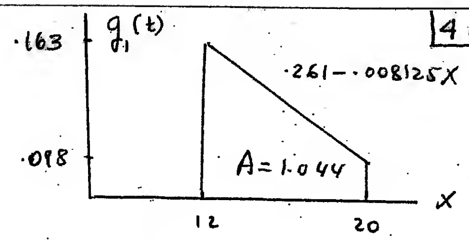
Thus, $K_2 = 99$

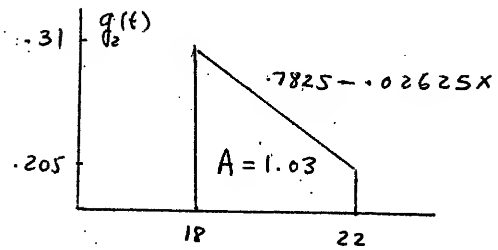
$f_1(t) = \dfrac{1.96}{t}$, $\quad 12 \le t \le 20$

$f_2(t) = \dfrac{99}{t^2}$, $\quad 18 \le t \le 22$

continued...

4 continued





$$h_1(t) = \frac{.261 - .008125t}{1.044}$$

$$= .25 - .007783t$$

$$H_1(t) = .025x - .00778\frac{x^2}{2}\Big|_{12}^{t}$$

$$= .25t - .003892t^2 - 2.44$$

$$h_2(t) = \frac{.7825 - .02625t}{1.03}$$

$$= .76 - .0255t$$

$$H_2(t) = .76t - .01275t^2 - 9.55$$

Sample computations from $H_2(t)$:

Step 1: $R_1 = .0589$

$$.76t - .01275t^2 - 9.55 = .0589$$

$$t^2 - 59.6t + 753.64 = 0$$

$$t = \frac{59.6 \pm \sqrt{(-59.6)^2 - 4 \times 753.64}}{2}$$

$$= 18.2$$

Step 2: $R = .6733$

Step 3: $$\frac{f_2(18.21)}{g_2(18.21)} = \frac{\left(\frac{99}{18.21^2}\right)}{.7825 - .02625 \times 18.21}$$

$$= .98 > .6733$$

Reject t.

continued...

Set 16.4a

1

Multiplicative Congruential Method

Input data	
b =	17
c =	111
u0 =	7
m =	103
How many numbers?	50
Output results	
Press to Generate Sequence	

Generated random numbers:

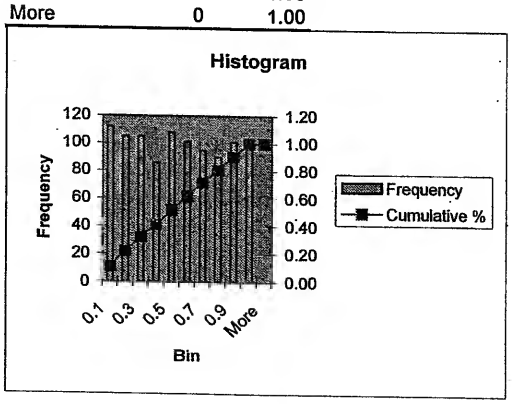
1	0.23301
2	0.03883
3	0.73786
4	0.62136
5	0.64078
6	0.97087
7	0.58252
8	0.98058
9	0.74757
10	0.78641
11	0.44660
12	0.66990
13	0.46602
14	0.00000
15	0.07767
16	0.39806
17	0.84466
18	0.43689
19	0.50485
20	0.66019
21	0.30097
22	0.19417
23	0.37864
24	0.51456
25	0.82524
26	0.10680
27	0.89320
28	0.26214
29	0.53398
30	0.15534
31	0.71845
32	0.29126
33	0.02913
34	0.57282
35	0.81553
36	0.94175
37	0.08738
38	0.56311
39	0.65049
40	0.13592
41	0.38835
42	0.67961
43	0.63107
44	0.80583
45	0.77670
46	0.28155
47	0.86408
48	0.76699
49	0.11650
50	0.05825

2

R=RAND()	Bin
0.813455	0.1
0.21757	0.2
0.937991	0.3
0.840823	0.4
0.19536	0.5
0.681599	0.6
0.829291	0.7
0.377723	0.8
0.149187	0.9
0.965781	1
0.808752	
0.957601	
0.502469	
0.620944	
0.992405	
0.97218	
0.051905	
0.144368	
0.129308	
0.676603	
0.140868	
0.486705	
0.12415	
0.821802	
0.954853	
0.301267	
0.827929	
0.917179	
0.07369	
0.462159	
0.333902	
0.390604	
0.723163	
0.041401	
0.805603	
0.556012	

Bin	*Frequency*	*umulative %*
0.1	112	0.11
0.2	105	0.22
0.3	105	0.32
0.4	86	0.41
0.5	108	0.52
0.6	101	0.62
0.7	95	0.71
0.8	90	0.80
0.9	101	0.90
1	97	1.00
More	0	1.00

Sample size = 1000



$C = 2$ barbers

$f_1(t) = .1e^{-.1t}, \quad t > 0$

$f_2(t) = \frac{1}{15}, \quad 15 \le t \le 30$

$t_1 = -12 \ln R$

$t_2 = 15 + 15R$

A_1 at $T = 0$:

$T(A_2) = 0 + (-10 \ln .0589) = 28.3$

$T(D_2) = 0 + (15 + 15 \times .6733) = 25.1$

Barber 1 busy

D_2 at $T = 25.1$:

Barber 1 idle

A_2 at $T = 28.3$:

$T(A_3) = 28.3 - 10 \ln .4799 = 35.6$

$T(D_2) = 28.3 + (15 + 15 \times .9486) = 57.5$

Barber 1 busy $\boxed{A_3 \quad D_2}$

A_3 at $T = 35.6$:

$T(A_4) = 35.6 - 10 \ln .6139 = 40.5$

$T(D_3) = 35.6 + (15 + 15 \times .5933) = 59.5$

Barber 2 busy $\boxed{A_4 \quad D_2 \quad D_3}$

A_4 at $T = 40.5$:

$T(A_5) = 40.5 - 10 \ln .9341 = 41.2$

A_4 waits in queue

$\boxed{A_5 \quad D_2 \quad D_3}$ $\boxed{A_4}$ ← queue

A_5 at $T = 41.2$:

$T(A_6) = 41.2 - 10 \ln .1782 = 58.4$

A_5 waits in queue

$\boxed{D_2 \quad A_6 \quad D_3}$ $\boxed{A_4 \quad A_5}$ ← queue

continued...

D_2 at $T = 57.5$:

Barber 1 idle

Take A_4 out of queue

$T(D_4) = 57.5 + 15 + 15 \times .3473 = 77.7$

Barber 1 busy

$\boxed{A_6 \quad D_3 \quad D_4}$ $\boxed{A_5}$ ← queue

A_6 at $T = 58.4$:

$T(A_7) = 58.4 - 10 \ln .5644 = 64.1$

Put A_6 in queue $\boxed{D_3 \quad A_7 \quad D_4}$

D_3 at $T = 59.5$: $\boxed{A_5 \quad A_6}$ ← queue

Barber 2 idle

Take A_5 out of queue

$T(D_5) = 59.5 + 15 + 15 \times .3529 = 79.8$

Barber 2 busy

$\boxed{A_7 \quad D_4 \quad D_5}$ $\boxed{A_6}$ ← queue

A_7 at $T = 64.1$:

$T(A_8) = 64.1 - 10 \ln .3646 = 74.2$

Put A_7 in queue

$\boxed{A_8 \quad D_4 \quad D_5}$ $\boxed{A_6 \quad A_7}$ ← queue

A_8 at $T = 74.2$:

$T(A_9) = 74.2 + (-10 \ln .7676)$

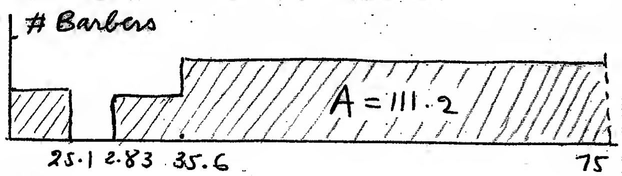
$= 76.8$

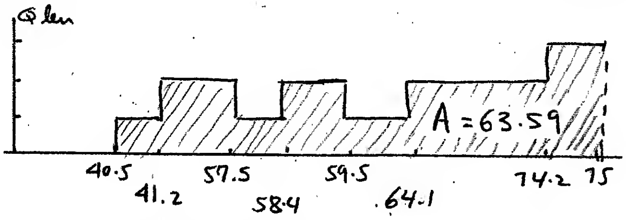
Place A_8 in queue.

$\boxed{A_9 \quad D_4 \quad D_5}$ $\boxed{A_6 \quad A_7 \quad A_8}$ ← queue

continued...

Set 16.5a





Av. facility utilization $= \frac{111.2}{75}$

$= 1.48$ barbers

Av. queue length $= \frac{63.59}{75} = .8$ customer

Av. waiting time in queue $= \frac{63.59}{8}$

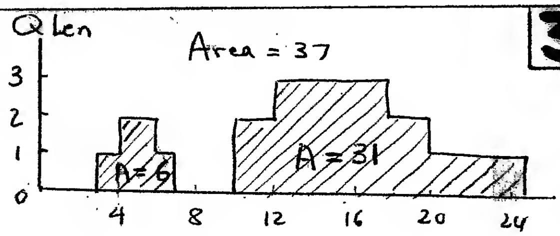
$= 7.95$ min

Av. waiting time for those who must wait $= \frac{63.59}{5} = 12.72$ min

2

(a) Observation.
(b) Time.
(c) Observation.
(d) Observation.
(e) Observation.
(f) Time.

3

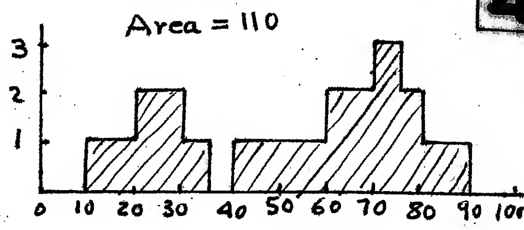


(a) $\bar{Q} = \frac{37}{25} = 1.48$ customers

(b) Number of waiting customers = 5

$\bar{W} = \frac{37}{5} = 7.4$ hours

4



(a) Average utilization

$= \frac{110}{100} = 1.1$ barber

(b) Average idle time

$= \frac{10 + (40-35) + (100-90)}{3}$

$= \frac{25}{3}$

$= 8.33$ minutes

I

Simulation of a Single-Server Queueing Model

Nbr of arrivals =		10	[redacted]				
Enter x in column A to select interarrival pdf:							
	Constant =						
x	Exponential:	λ =	0.0667				
	Uniform:	a =	8	b =	9		
	Triangular:	a =		b =		c =	
Enter x in column A to select service time pdf:							
	Constant =						
	Exponential:	μ =					
x	Uniform:	a =	10	b =	15		
	Triangular:	a =		b =		c =	

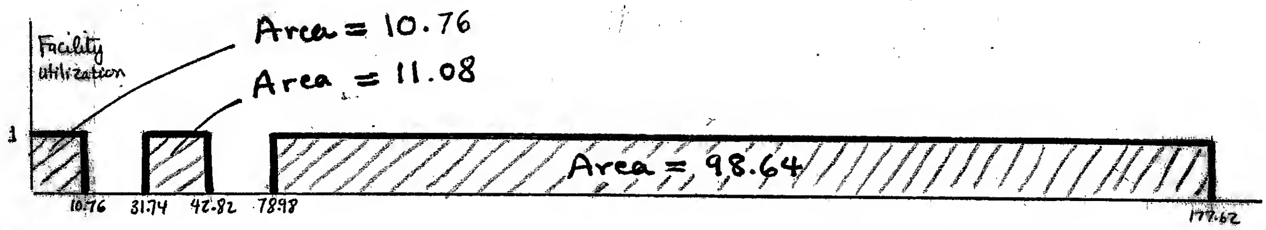
Output Summary

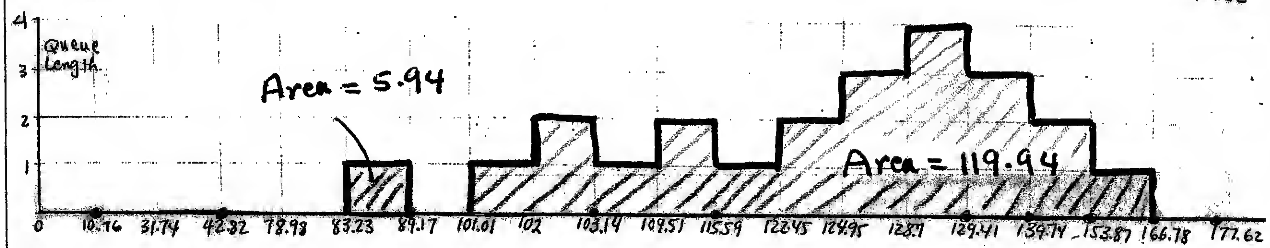
Av. facility utilization =	0.68
Percent idleness (%) =	32.17
Maximum queue length=	4
Av. queue length, Lq =	0.71
Av. nbr in system, Ls =	1.39
Av. queue time, Wq =	12.58
Av. system time, Ws =	24.63
Sum(ServiceTime) =	120.47
Sum(Wq) =	125.85
Sum(Ws) =	246.32

Press F9 to trigger a new simulation run.

Simulation Calculations

Nbr	InterArvlTime	ServiceTime	ArrvlTime	DepartTime	Wq	Ws
1	31.74	10.76	0.00	10.76	0.00	10.76
2	47.24	11.07	31.74	42.82	0.00	11.07
3	4.25	10.19	78.98	89.17	0.00	10.19
4	17.78	13.96	83.23	103.14	5.94	19.91
5	0.99	12.45	101.01	115.59	2.13	14.58
6	7.51	13.82	102.00	129.41	13.59	27.41
7	12.94	10.33	109.51	139.74	19.90	30.23
8	2.51	14.13	122.45	153.87	17.29	31.42
9	3.74	12.90	124.95	166.78	28.92	41.82
10	9.02	10.84	128.70	177.62	38.08	48.92





From the graph:

$$\sum \text{service times} = 10.76 + 11.08 + 98.64 = 120.48$$

$$\sum \text{queue waiting times} = 5.94 + 119.94 = 125.88$$

(The small difference between these answers and the simulation output is because of roundoff error.)

$$\text{Av. facility utilization} = \frac{120.48}{177.62} = .6783$$

$$\text{Av. queue length} = \frac{125.88}{177.62} = .7087$$

$$\text{Av. waiting time in queue} = \frac{125.88}{10} = 12.588$$

$$\text{Av. waiting time in system} = \frac{120.48 + 125.88}{10} = 24.636$$

Set 16.5b

2

Nbr of arrivals =		500	<<Maximum 5	
Enter x in column A to select interarri				
	Constant =			
x	Exponential:	λ =	4	
	Uniform:	a =		b =
	Triangular:	a =		b =
Enter x in column A to select service t				
	Constant =			
x	Exponential:	μ =	6	
	Uniform:	a =		b =
	Triangular:	a =		b =
			Output Summar	

Run		
(1)	Av. facility utilization =	0.66
	Percent idleness (%) =	33.84
	Maximum queue length=	0
	Av. queue length, Lq =	1.42
	Av. nbr in system, Ls =	2.08
	Av. queue time, Wq =	0.37
	Av. system time, Ws =	0.54
(2)	Av. facility utilization =	0.61
	Percent idleness (%) =	38.65
	Maximum queue length=	0
	Av. queue length, Lq =	0.91
	Av. nbr in system, Ls =	1.52
	Av. queue time, Wq =	0.24
	Av. system time, Ws =	0.40
(3)	Av. facility utilization =	0.65
	Percent idleness (%) =	35.11
	Maximum queue length=	0
	Av. queue length, Lq =	0.91
	Av. nbr in system, Ls =	1.56
	Av. queue time, Wq =	0.22
	Av. system time, Ws =	0.38
(4)	Av. facility utilization =	0.68
	Percent idleness (%) =	31.70
	Maximum queue length=	0
	Av. queue length, Lq =	1.35
	Av. nbr in system, Ls =	2.03
	Av. queue time, Wq =	0.32
	Av. system time, Ws =	0.48
(5)	Av. facility utilization =	0.60
	Percent idleness (%) =	39.83
	Maximum queue length=	0
	Av. queue length, Lq =	1.14
	Av. nbr in system, Ls =	1.74
	Av. queue time, Wq =	0.30
	Av. system time, Ws =	0.46

continued...

Summary:

	utiliz	Lq	Ls	Wq	Wq
mean	.64	1.146	1.786	.29	.452
Std. Dev.	.0339	.2388	.2598	.0608	.0642

95% confidence limits:

$t_{4,.025} = 2.776$

$UCL = \bar{X} + \frac{2.776\,S}{\sqrt{n}} = \bar{X} + 1.245$

$LCL = \bar{X} - 1.245$

	utiliz	Lq	Ls	Wq	Ws
LCL	.598	.850	1.464	.215	.372
UCL	.682	1.442	2.108	.365	.531

Poisson queue output:

Scenario 1-- (M/M/1):(GD/infinity/infinity)

Lambda =	4.00000	Mu =	6.00000
Lambda eff =	4.00000	Rho/c =	0.66667
Ls =	2.00000	Lq =	1.33333
Ws =	0.50000	Wq =	0.33333

3

s =	200	<<Maximum 500			
umn A to select interarrival pdf:					
=	11.5				
ial:	λ =				
	a =		b =		
r:	a =		b =		c =
umn A to select service time pdf:					
=					
ial:	μ =				
	a =		b =		
r:	a =	9	b =	9.5	c = 15

Run		
(1)	Av. facility utilization =	0.96
	Percent idleness (%) =	4.20
	Maximum queue length=	2
	Av. queue length, Lq =	0.12
	Av. nbr in system, Ls =	1.08
	Av. queue time, Wq =	1.36
	Av. system time, Ws =	12.38

continued...

(2)

Av. facility utilization =	0.96
Percent idleness (%) =	3.85
Maximum queue length=	2
Av. queue length, Lq =	0.12
Av. nbr in system, Ls =	1.08
Av. queue time, Wq =	1.33
Av. system time, Ws =	12.39

(3)

Av. facility utilization =	0.97
Percent idleness (%) =	2.98
Maximum queue length=	2
Av. queue length, Lq =	0.19
Av. nbr in system, Ls =	1.16
Av. queue time, Wq =	2.14
Av. system time, Ws =	13.33

(4)

Av. facility utilization =	0.96
Percent idleness (%) =	3.58
Maximum queue length=	2
Av. queue length, Lq =	0.16
Av. nbr in system, Ls =	1.13
Av. queue time, Wq =	1.88
Av. system time, Ws =	12.97

(5)

Av. facility utilization =	0.97
Percent idleness (%) =	3.39
Maximum queue length=	2
Av. queue length, Lq =	0.17
Av. nbr in system, Ls =	1.14
Av. queue time, Wq =	2.00
Av. system time, Ws =	13.12

utilization:

$$\text{mean} = \frac{.96+.96+.97+.96+.97}{5} = .964$$

$$\text{St. dev.} = .0311$$

Set 16.6a

1

$W_1 = \frac{14}{3} = 4.67$ (time units)

$W_2 = \frac{10}{4} = 2.5$

$W_3 = \frac{11}{3} = 3.67$

$W_4 = \frac{6}{3} = 2$

$W_5 = \frac{15}{4} = 3.75$

$\bar{W} = \frac{4.67+2.5+3.67+2+3.75}{5}$

$= 3.32$ time units

2

Discard observations during the transient period $(0, 100)$

$W_1 = \frac{12+30+10+14+16}{5} = 16.4$ time units

$W_2 = \frac{15+17+20+22}{4} = 18.5$

$W_3 = \frac{10+20+30+15+25+31}{6} = 21.83$

$W_4 = \frac{15+17+20+14+13}{5} = 15.8$

$W_5 = \frac{25+30+15}{3} = 23.33$

$\bar{W} = 19.17 \qquad S = 3.3$

Confidence interval

$\bar{W} \pm t_{.025,4}\frac{S}{\sqrt{n}}$

$= 19.17 \pm 2.776\frac{3.3}{\sqrt{5}}$

or

$15.07 \le \mu \le 23.27$

3

Batch	a_i	b_i	y_i
1	6	7	.869
2	10	7	1.369
3	6	9	.584
	$\bar{a} = 7.33$	$\bar{b} = 7.67$	$\bar{y} = .941$
			$S_y = .397$

continued...

3 continued

$y_i = \frac{3\times 7.33}{7.67} - \frac{(3-1)(3\times 7.33 - a_i)}{3\times 7.67 - b_i}$

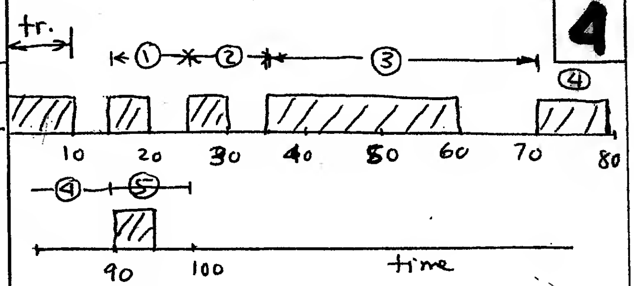
$= 2.867 - \frac{43.98 - 2a_i}{23.01 - b_i}$

95% confidence interval:

$.941 - 2.776\frac{.397}{\sqrt{3}} \le \mu \le .941 + 2.776\frac{.397}{\sqrt{3}}$

$.305 \le \mu \le 1.577$

4



(a) Start points are 15, 25, 35, 70, 90

(b)

Batch	a_i	b_i	y_i
1	5	10	.54
2	5	10	.54
3	25	35	.94
4	10	20	.45
5	5	10	.54
	$\bar{a} = 10$	17	$\bar{y} = .602$
			$S_y = .193$

$y_i = \frac{5\times 10}{17} - \frac{4(5\times 10 - a_i)}{5\times 17 - b_i}$

$= 2.94 - \frac{200 - 4a_i}{85 - b_i}$

$.602 - 2.776\frac{.193}{\sqrt{5}} \le \mu \le .602 + 2.776\frac{.193}{\sqrt{5}}$

or $\quad .36 \le \mu \le .84$

(c) $t = \frac{90}{5} = 18$

i	1	2	3	4	5
A	8	13	14	10	5
u_i	.44	.72	.78	.56	.28

Mean = .556, Std. Dev. = .2042